

Efficient data-driven strategy

for 3D model-preconditioning FWI

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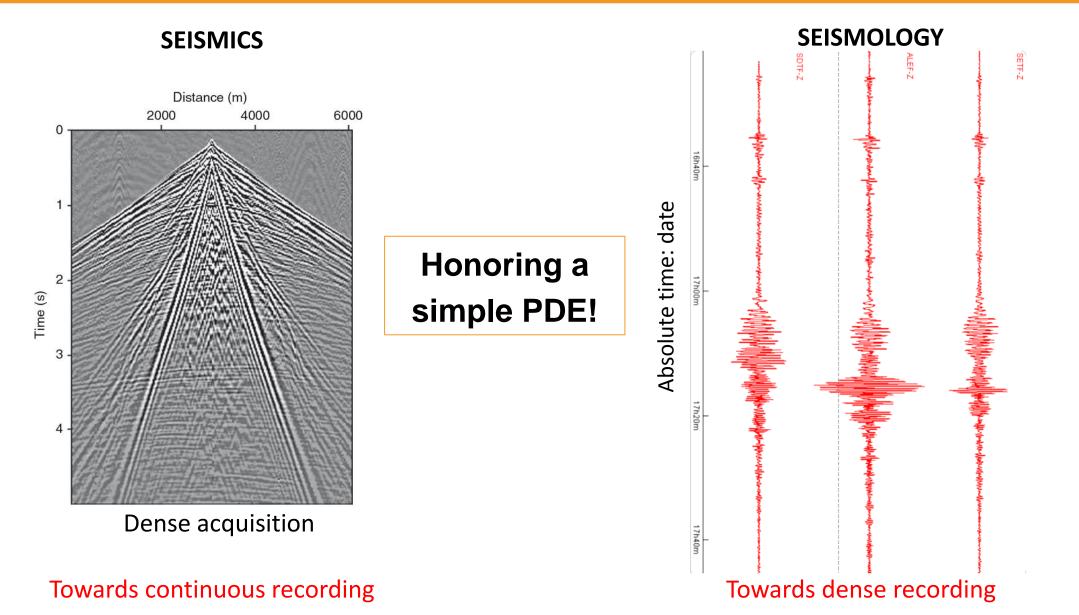
* Now at Chevron Australia

http://seiscope2.osug.fr



Seismic data







1. Motivation

2. FWI: single scattering

3. PDE visco-elastic wave propagation

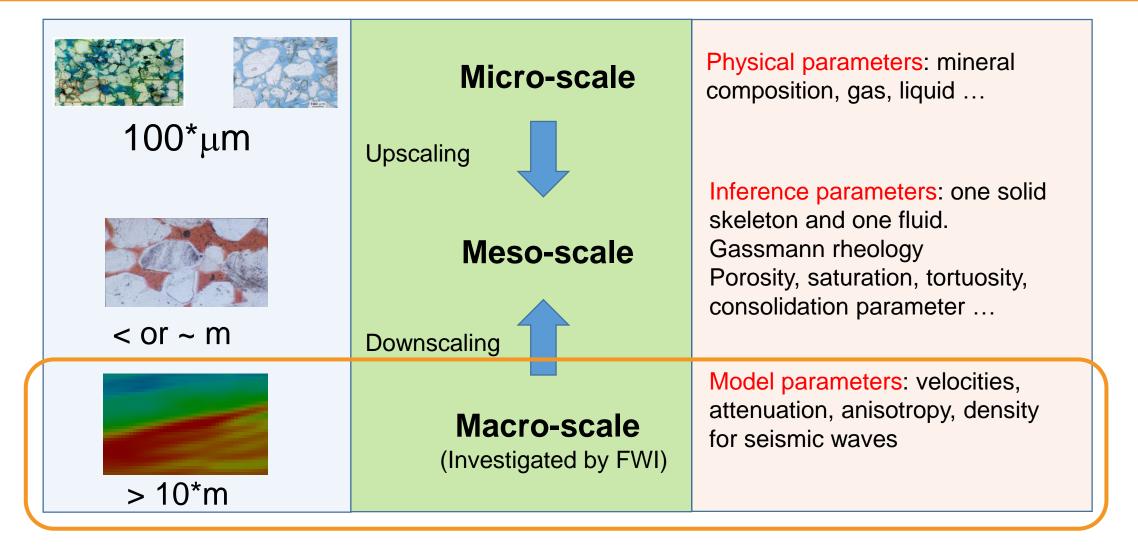
4. Model discretization & preconditioning

5.3D elastic SEAM II Foothills application

Ebook of SEG: encyclopedia of exploration geophysics http://library.seg.org/doi/abs/10.1190/1.9781560803027.entry6

Model/Physical parameter hunting?





Important parameters at the macro-scale level ?

Attenuation, Elasticity, Anisotropy, Density

High-resolution seismic imaging



> Macro-scale imaging: FWI provides high-resolution capacity

- Vertical components or 4C data
- Body waves versus surface waves
- Diving waves versus reflected waves

> Which physics to consider at this scale?

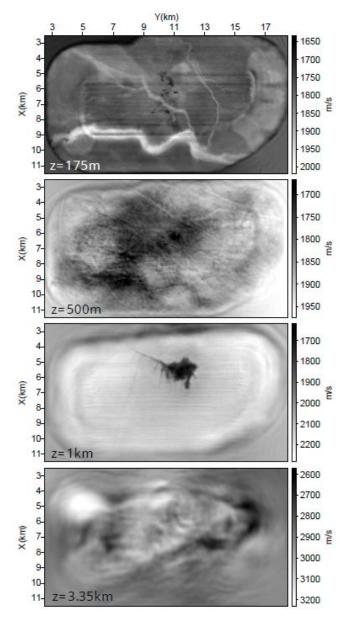
- Visco-elastic anisotropic propagation
- Related model parameters ...

> Medium interpretation: which physics to consider?

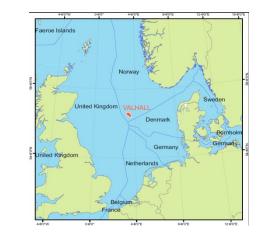
- Downscaling using biphasic model (Gassmann relation)
- Upscaling from multi-phases rock description related to physical parameters ...
- Inference step between downscaling and upscaling

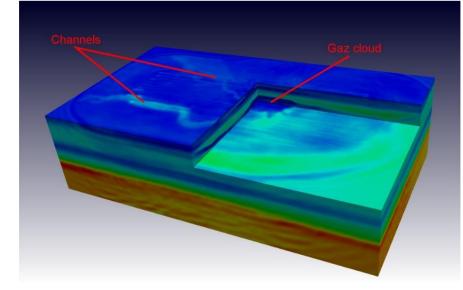
Macro-scale imaging

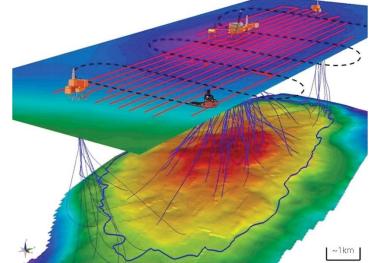




FWI provides high-resolution capacity







Operto & Miniussi (2017)

High-resolution seismic imaging



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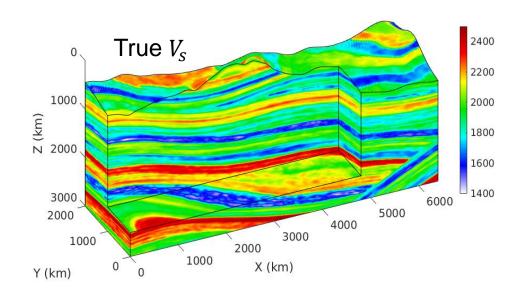
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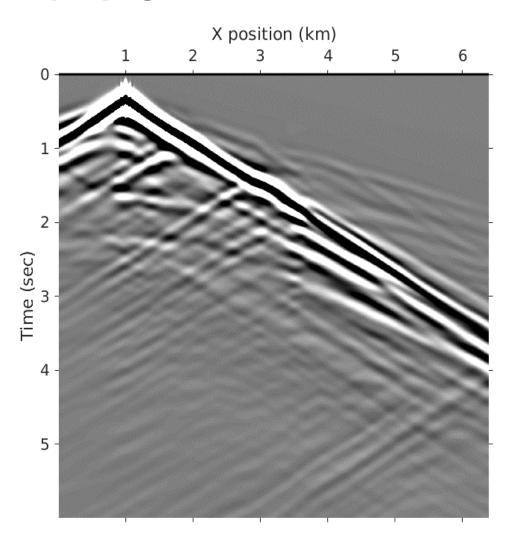
Which physics to consider at macro-scale?



Anisotropic visco-elastic propagation



- Highly dispersive surface waves
- Waves conversion P-S, body-surface
- Transmission/Reflection regimes
- Back-scattering due to steep slopes at the free surface



High-resolution seismic imaging



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> Medium interpretation: which physics to consider?

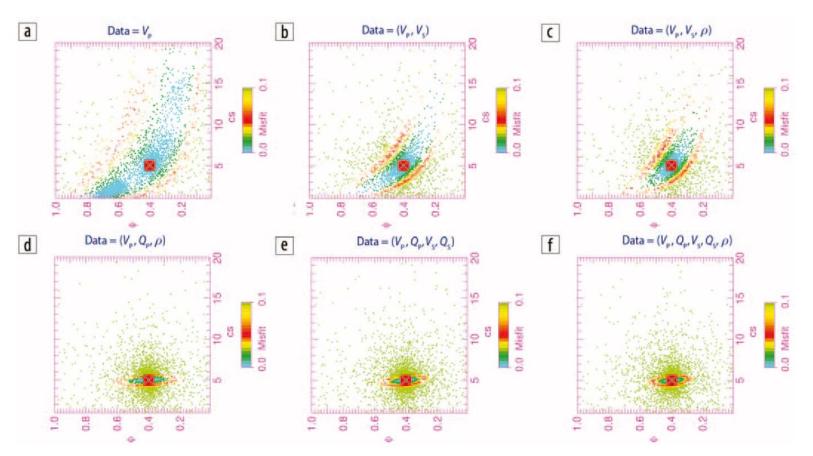
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- Inference step between downscaling and upscaling

\Rightarrow Towards reservoir interpretation and monitoring

Which physics to consider?



Physical interpretation = Many model parameters?



Model parameters are now the data used for downscaling ...

> Pride (2005); Chopra & Marfurt (2007); Mavko et al. (2009); Dupuy et al. (2016)

Gassmann's equation: porosity ϕ and consolidation parameter c_s



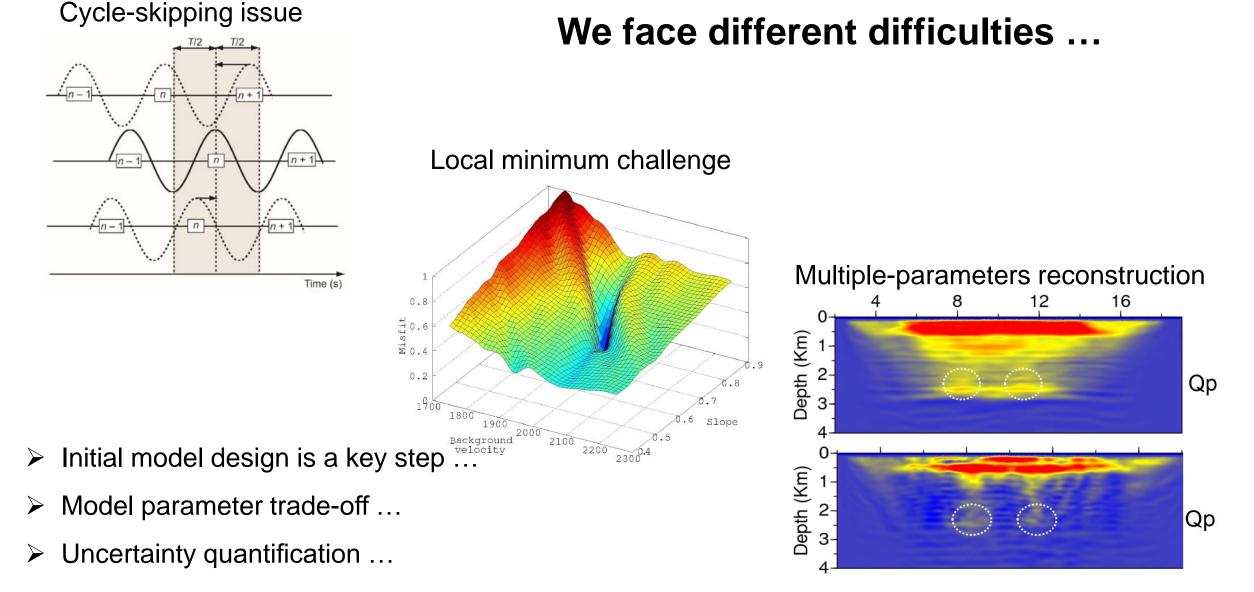
Model parameters reconstruction

FWI pros and cons

Non-linearity of FWI

High-resolution seismic imaging





Nov 6-10



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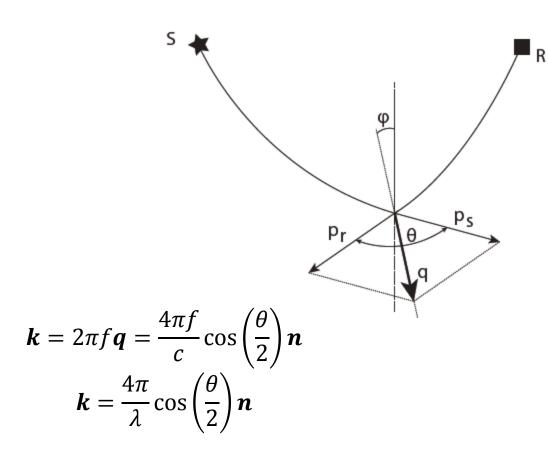
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FWI = simple wave-matter interaction



(Devaney, 1982)



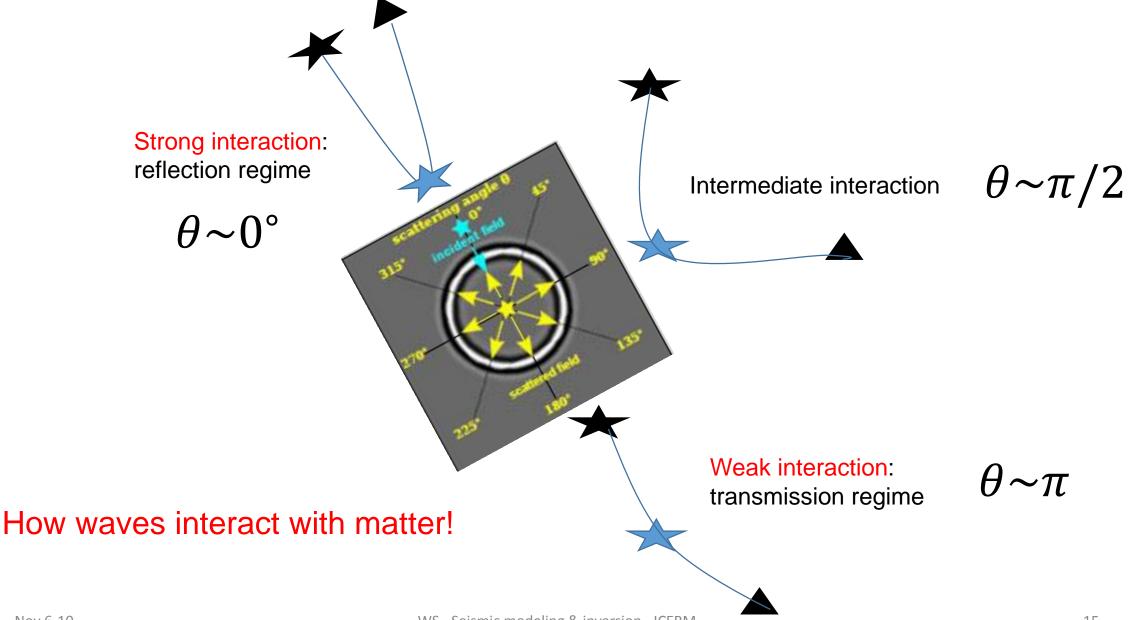
- FWI is an ill-posed problem based on a single-scattering formulation
- Model is described through a pixel structure (# from a blocky structure)
- The model wavenumber spectrum is probed through this pixel strategy
 - f Frequency
 - θ Aperture or illumination angle

Controlling parameters of the model velocity spectrum

Low k – low frequency f or aperture angle θ around π (weak interaction) High k – high frequency f or aperture angle θ around 0 (strong interaction)

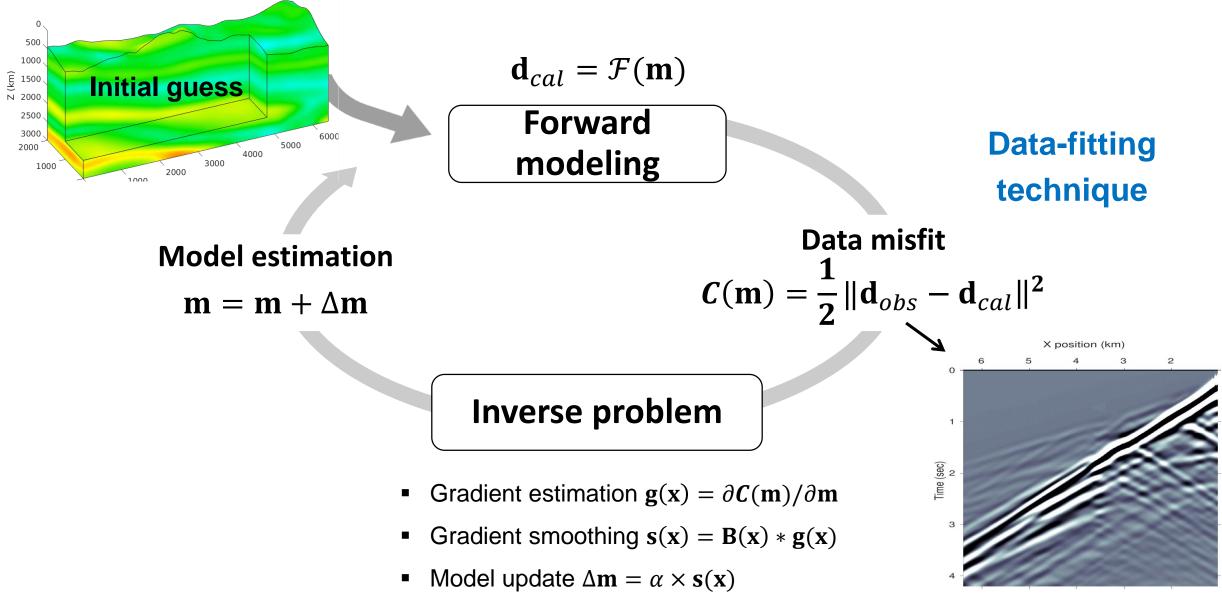
Scattering diagram





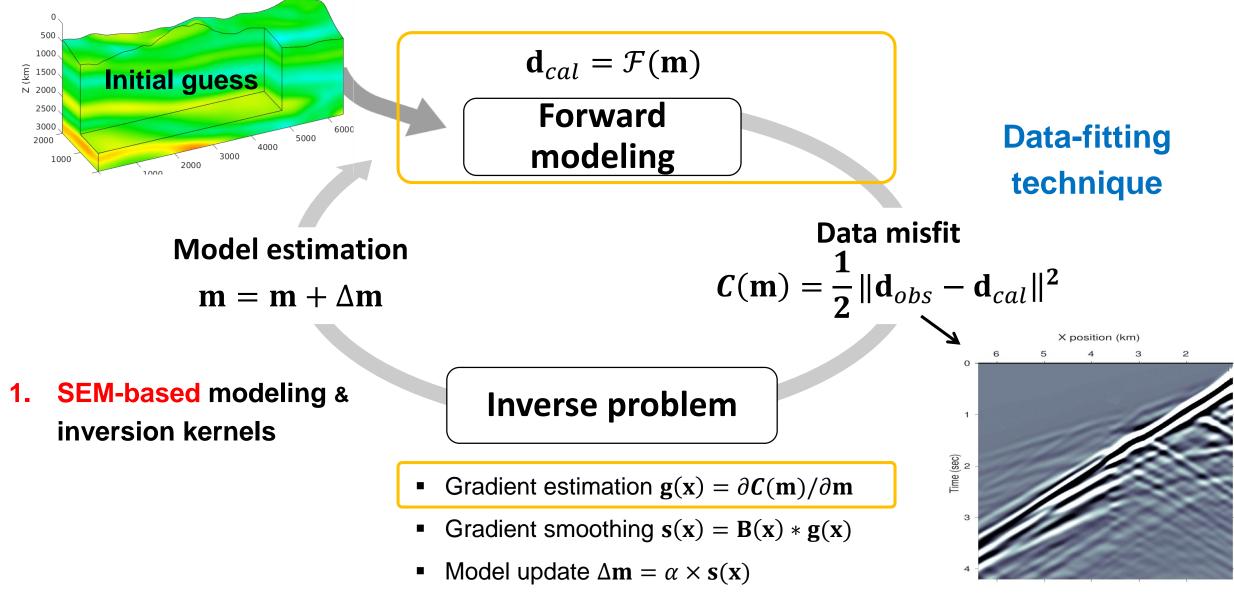
FWI strategy





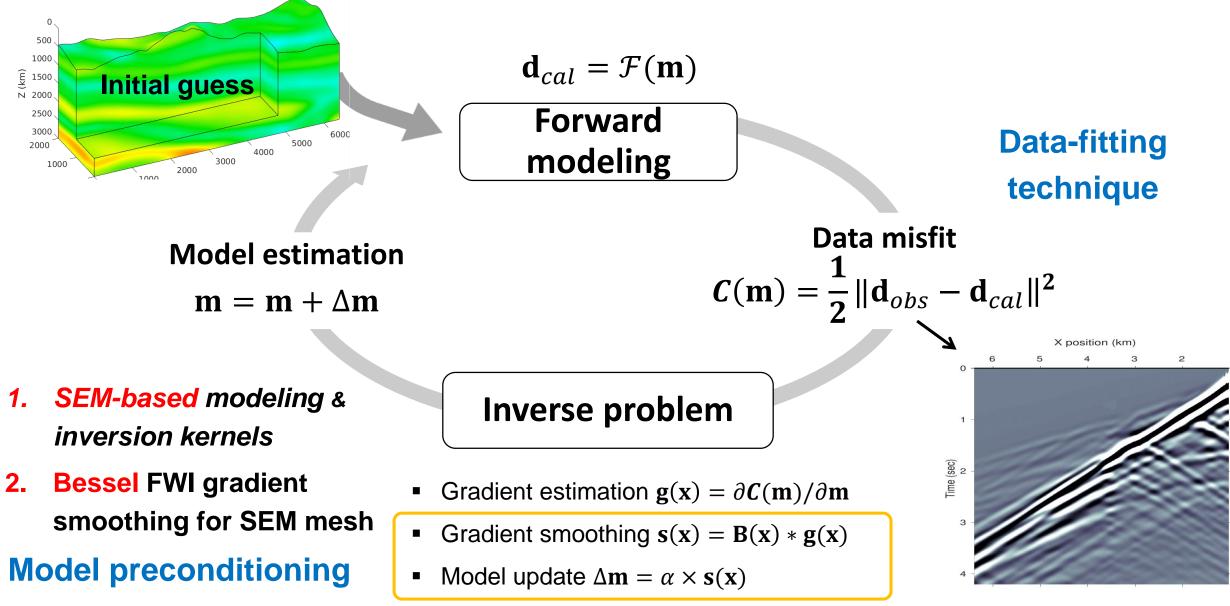
FWI strategy





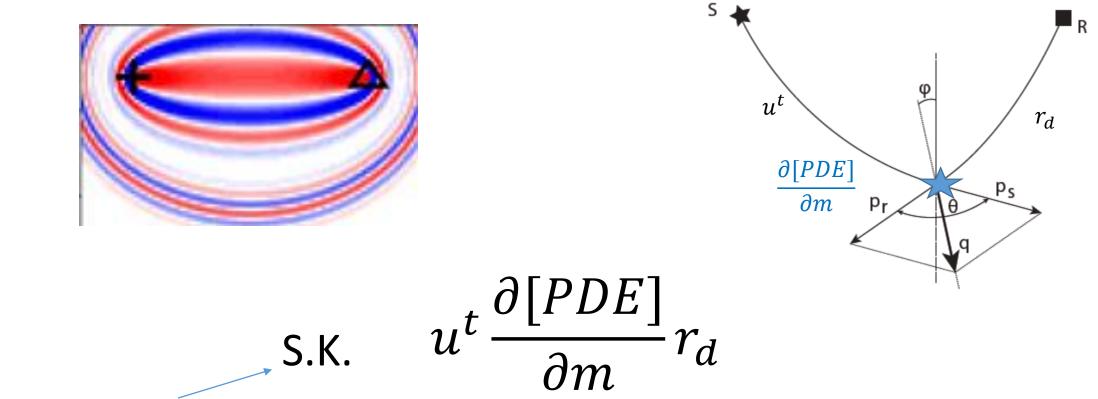
FWI strategy





FWI gradient: often all you need





Sensitivity kernel

Zero-lag cross-correlation of incident u^t and adjoint r_d fields through interlaced backward-incident and adjoint integration



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Integrated approach: FWI design should not be reduced to wave propagation design

Complex topography

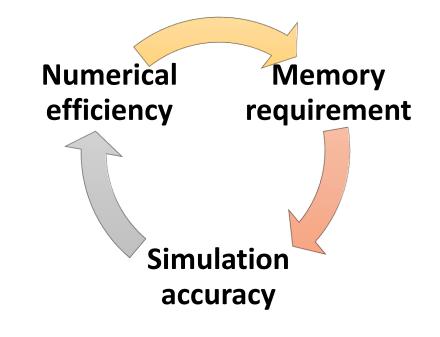
- Simple geometry representation.
- Accurate boundary free-surface conditions.

3D (visco)elastic modeling & FWI

- Complete and accurate physics seen by waves
- Simultaneous design of modeling/adjoint/gradient

Time-domain

- Signal muting and multi-frequencies processing
- Data-component hierarchy FWI, thanks to the causality



Attenuation: Efficient implementation



5



- > Acoustic might not be enough!
- Elastic neither: Attenuation is required when fitting phase & amplitude!

Visco-elastic 3D aniso-elastic reconstruction

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- ✓ Tarantola (1988): **Convolutional rheology** with application by Charara et al. (2000) ⇒ **Computationally intensive.**
- ✓ Tromp (2005) & Liu and Tromp (2006): General multiparameter workflow with adjoint methods.
- ✓ Fichtner & van Driel (2014): Clarification of the **Q parameter imaging** of Tromp (2005) ⇒ Lowering the computational needs.
- ✓ Yang et al (2016): Explicit formulations for FWI gradients using visco-anisotropic elastic wave propagation based on standard linear solid (SLS) mechanisms

 \Rightarrow Straightforward numerical implementation.

Wave propagation: lossy medium



Time domain

$$\begin{cases} \rho \partial_{tt} \mathbf{u} = D\boldsymbol{\sigma} + \mathbf{f} \\ \boldsymbol{\varepsilon} = D^{t} \mathbf{u} \\ \boldsymbol{\sigma} = C\boldsymbol{\varepsilon} - C^{R} \sum_{l=1}^{L} \boldsymbol{\psi}_{l} + \boldsymbol{\mathcal{T}} \\ \partial_{t} \boldsymbol{\psi}_{l} + \omega_{l} \boldsymbol{\psi}_{l} = \omega_{l} y_{l} \boldsymbol{\varepsilon}, \\ l = 1, \dots, L \end{cases}$$

Wavefield conditions

- Medium at rest at initial time (zero initial conditions)
- Unbounded domain (free surface condition -zero stress- and absorbing boundary conditions)

Standard Linear Solid: Generalized Maxwell model or Generalized Zener model

• Attenuation is carried by *L* sets of **memory variables** ψ_l = non-physical parameters.

Quantities ω_l and y_l are uniform inside the medium (resonance frequencies and relative weights)

- Attenuation = SLS Q-constant approx. over frequencies.
- Memory variables obey a 1st order equation.

Additional needs: storing decimated boundaries (inside nearby PML)

and few snapshots for backpropagation

(Yang at al, 2016a, 2016b)

Wave propagation: FWD modeling OK



Time domain

 $\begin{cases} \rho \partial_{tt} \mathbf{u} = D\boldsymbol{\sigma} + \mathbf{f} \\ \boldsymbol{\varepsilon} = D^{t} \mathbf{u} \\ \boldsymbol{\sigma} = C\boldsymbol{\varepsilon} - C^{R} \sum_{l=1}^{L} \boldsymbol{\psi}_{l} + \boldsymbol{\mathcal{T}} \\ \partial_{t} \boldsymbol{\psi}_{l} + \omega_{l} \boldsymbol{\psi}_{l} = \omega_{l} y_{l} \boldsymbol{\varepsilon}, \\ l = 1, \dots, L \end{cases}$

Heterogeneities inside the medium described by

- C Unrelaxed (elastic) stiffness tensor (anisotropic);
- C^{R} Relaxed stiffness tensor (isotropic);

Relaxed « Lamé »	$\lambda^{R} + 2\mu^{R} = \frac{1}{2} O^{-1} \sum_{k=1}^{3} C_{kk}$	$u^{R} = \frac{1}{2} O^{-1} \sum_{k=1}^{6} C_{kk}$
coefficients:	$\chi^{\mu} + 2\mu^{\mu} = \frac{1}{3}Q_p - \sum_{i=1}^{n} C_{ii};$	$\mu^{\mu} = 3 Q_s \sum_{j=4}^{2} C_{jj}$

Elastic system is conservative: self-adjoint structure of PDE

 $\rho \partial_{tt} \mathbf{u} = DCD^t \mathbf{u} + \mathbf{f} \implies$ Stable backpropagation of the wavefield.

> With attenuation, the system is no more conservative!

 $\rho \partial_{tt} \mathbf{u} = DCD^t \mathbf{u} - DC^R \sum_{l=1}^{L} \psi_l + \mathbf{f} \implies \text{Unstable backpropagation of the wavefield!}$

Tracking the total energy for detecting the instability during the backpropagation: if divergence is observed, use stored snapshots to restart the backpropagation from them (assisted checkpointing strategy)

Incident + Adjoint propagation



Incident field

$$\begin{cases} \rho \partial_{tt} \mathbf{u} = DCD^{t} \mathbf{u} - DC^{R} \sum_{s=1}^{L} \boldsymbol{\psi}_{s} + \mathbf{S} \\ \partial_{t} \boldsymbol{\psi}_{s} + \omega_{s} \boldsymbol{\psi}_{s} = \omega_{s} y_{s} D^{t} \mathbf{u} \end{cases}$$

u – Displacement;

 $\boldsymbol{\psi}_s$ – Memory variables;

S – Source term;

Adjoint field

$$\begin{cases} \rho \partial_{tt} \overline{\mathbf{u}} = DCD^{t} \overline{\mathbf{u}} - DC^{R} \sum_{s=1}^{L} \overline{\boldsymbol{\psi}_{s}} - R^{\dagger} \Delta d_{\mathbf{u}} \\ \partial_{t} \overline{\boldsymbol{\psi}_{s}} - \omega_{s} \overline{\boldsymbol{\psi}_{s}} = -\omega_{s} y_{s} D^{t} \overline{\mathbf{u}} \end{cases} \end{cases}$$

 $\overline{\mathbf{u}}$ – Displacement; $\overline{\boldsymbol{\psi}_s}$ – Memory variables; $\Delta d_{\mathbf{u}}$ – Data residual;

using Lagrange formulation (final and boundary conditions!)

- Similar but not identical structure and equations for incident and adjoint fields
- Computing incident field from initial time with zero initial conditions ③
- Computing adjoint field from final time with zero final conditions ③

+ recomputing incident field backward 🙁 but 😊

Inversion workflow

• Least-squares norm:

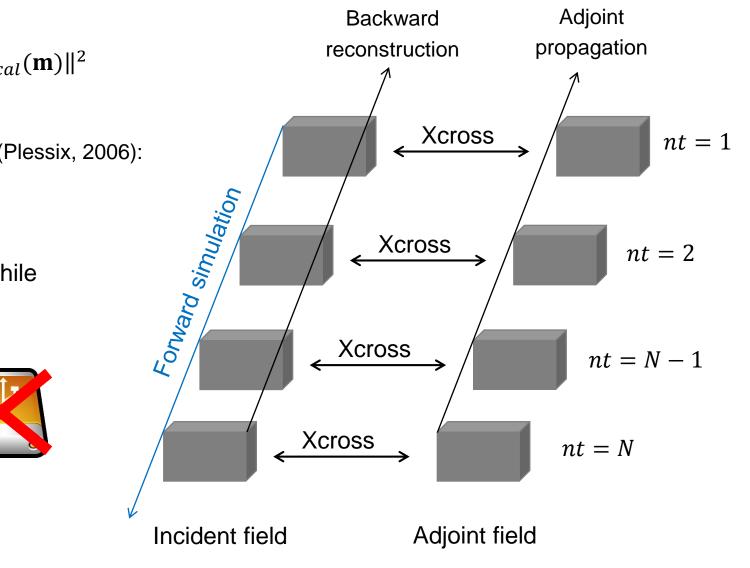
$$\frac{1}{2} \|\mathbf{d}_{obs} - \mathbf{d}_{cal}(\mathbf{m})\|^2$$

• All gradients = **Adjoint-state approach** (Plessix, 2006):

Directly accumulated during the **backpropagation** of the incident field while computing adjoint fields.

 \Rightarrow No I/O

Affordable numerical cost



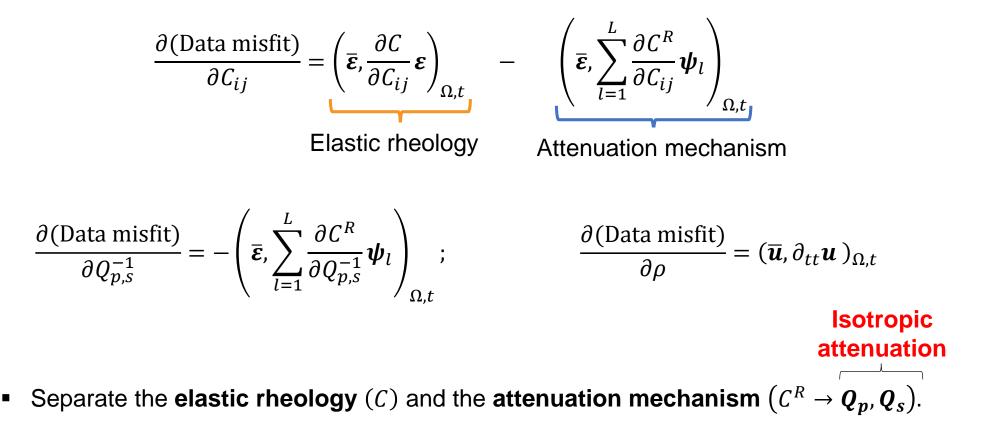
Acoustic case (Yang et al., 2016c)

SEISCOPE



\Rightarrow Attenuation affects the velocity estimation

• L₂ FWI gradient:



Anisotropic attenuation: VSP data?

SEM46 for modeling and inversion

Time-domain Spectral Element Method

SEM-based implementation

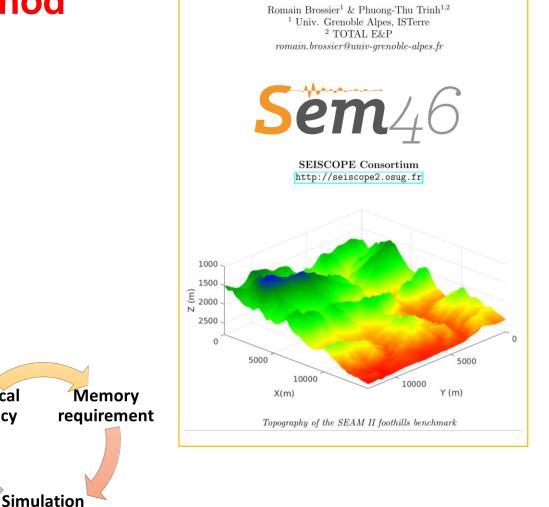
- Topography & simple geometry representation.
- Accurate boundary free-surface conditions.

3D (visco)elastic modeling & FWI

- Complete and accurate physics seen by waves.
- Simultaneous design of modeling/adjoint/gradient.

Time-domain

- Signal muting and multi-frequencies processing.
- Data-component hierarchy FWI (causality).



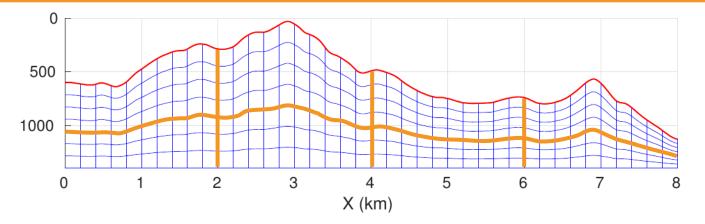
Numerical

efficiency

accuracy

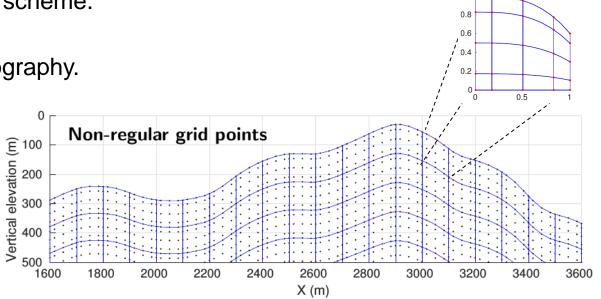
Cartesian-based deformed mesh





- Combine the accuracy of FE mesh with the easiness of implementation of FD grid.
- Avoid the heavy searching operator over the global mesh.
- Efficient domain-decomposition in a parallel scheme.
- Vertical deformed elements to follow the topography.
- High-order presentation of the topography

Numerical cost vs. simulation accuracy



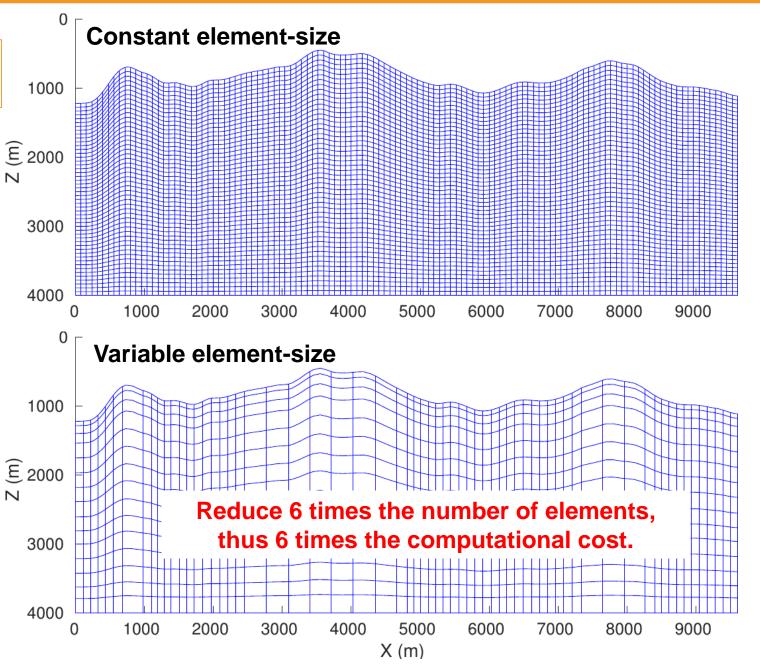
Variable element-size for modeling



Mesh design is constrained by ≥ **5 GLL points /min (wavelength)**

Same element-size everywhere.

- > Respect the theoretical resolution of FWI (0. $5\lambda_s$).
- > Follow the velocity variation.



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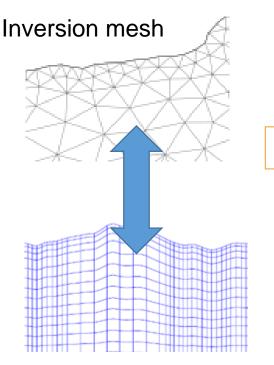
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Model discretization



Pixel-oriented FWI: which sampling strategy for this ill-posed problem ?



Model meshing adapted to the expected FWI resolution (few λ s).

Expensive back and forth projections, especially in 3D

An alternatrive could be the ROM strategy

Modeling meshing adapted to the required local sampling of wavelengths for wave propagation (fractions of λ)

- Same mesh for forward/inverse problems \Rightarrow Efficient computation.
- Mathematically ill-posed features of FWI: expected low-wavenumber content.
- **Preconditioning** and/or regularization is mandatory in FWI.

Necessary of preconditioning & regularization

Smoothing the FWI gradient!

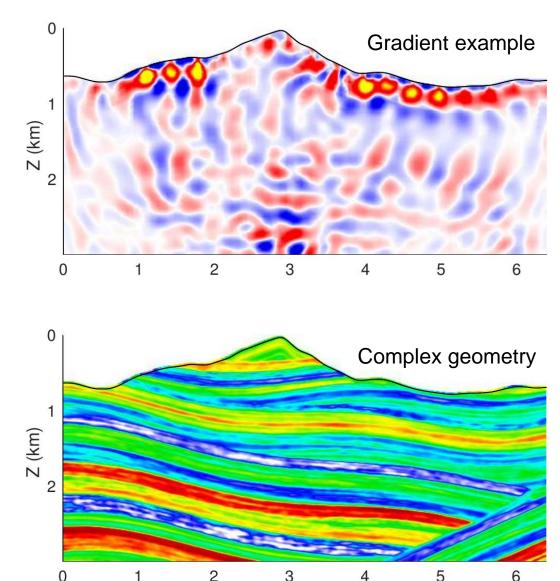
Why?

- > Suppress high-wavenumber artifacts
 - Acquisition footprints
 - Poor illumination
- Guide the inversion towards a desired solution

Need?

- Nonstationary & anisotropic operator
 - Anisotropic coherent lengths
 - Local 3D rotation
- Numerical efficiency
- SEM mesh compatible: Non-regular grid points

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X (km)

Bessel smoothing for SEM mesh

- Considering the sparse inverse operator: ٠
- 0° rotation, homogeneous ٠ coherent lengths:

$$\left[1 - \left(L_z^2 \frac{\partial^2}{\partial z^2} + L_x^2 \frac{\partial^2}{\partial x^2} + L_y^2 \frac{\partial^2}{\partial y^2}\right)\right] \mathbf{s}(\mathbf{x}) = \mathbf{g}(\mathbf{x})$$

 $B_{3D}^{-1}({\bf x})$

Smoothed

gradient

* $\widetilde{\mathbf{s}(\mathbf{x})} = \widetilde{\mathbf{g}(\mathbf{x})}$

Fully **anisotropic** & **nonstationary** filter:

→ Self-adjoint PDE

✓ Variable coherent lengths \checkmark 3D rotation and angles

 $\mathbf{P}(\mathbf{x}) = \begin{bmatrix} L_v \cos\varphi & L_u \sin\varphi & 0\\ -L_v \cos\theta \sin\varphi & L_u \cos\theta \cos\varphi & L_w \sin\theta\\ L_v \sin\theta \sin\varphi & -L_u \sin\theta \cos\varphi & L_w \cos\theta \end{bmatrix}$

Geological prior information

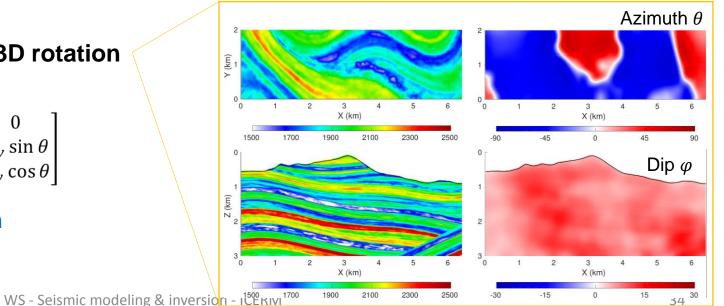
(Trinh et al, 2017b; Wellington et al, 2017)

$$\left[1 - \nabla_{z,x,y}^{t} \mathbf{P}(\mathbf{x}) \mathbf{P}^{t}(\mathbf{x}) \nabla_{z,x,y}\right] \mathbf{s}(\mathbf{x}) = \mathbf{g}(\mathbf{x}) \qquad \nabla_{z,x}$$

Raw

gradient

$$\nabla_{z,x,y} = \left(\partial_z, \partial_x, \partial_y\right)^t$$





Parallel implementation



$$\left[1 - \nabla_{z,x,y}^{t} \mathbf{P}(\mathbf{x}) \mathbf{P}^{t}(\mathbf{x}) \nabla_{z,x,y}\right] \mathbf{s}(\mathbf{x}) = \mathbf{g}(\mathbf{x})$$

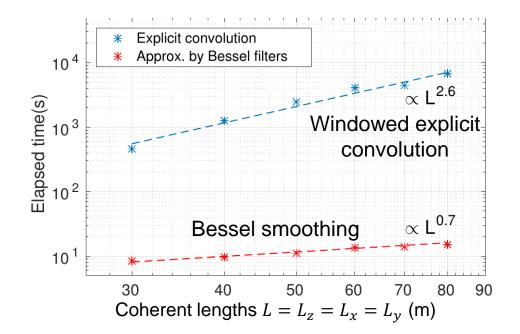
 $\mathbf{As} = \mathbf{g}$

• Self-adjoint PDE = Symmetric, well-conditioned, positive-definite linear system

⇒ Efficiently solved by a matrix-free parallel conjugate-gradient

- Linear numerical complexity O(Coherent length)
 In FD scheme: as cheap as tensorized Gaussian
 convolution.
- Smoothing \approx 0.4 % cost of 1 iteration

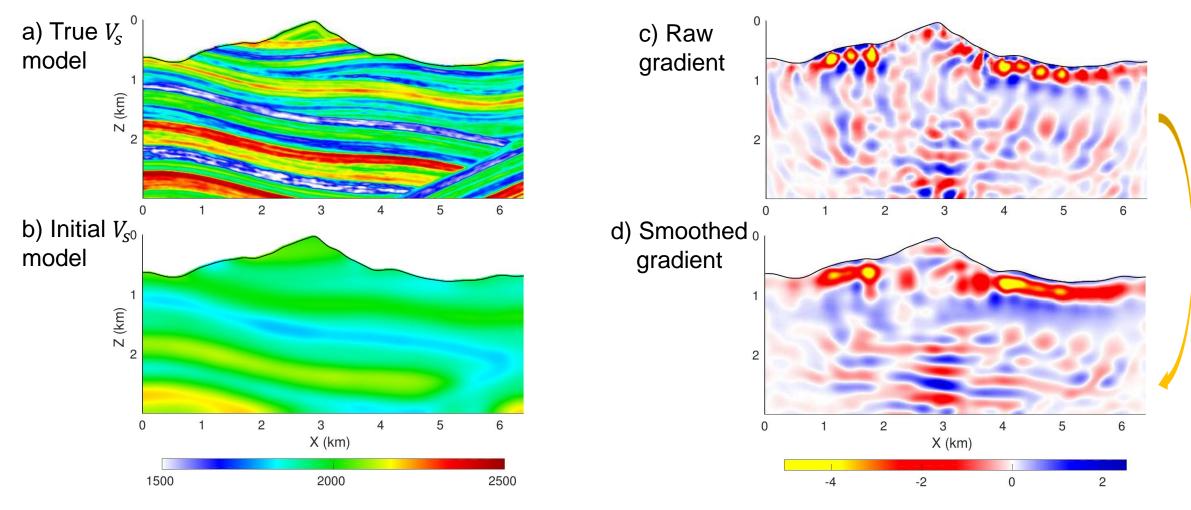
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Structure-oriented preconditioning



Nonstationary & anisotropic Bessel gradient preconditioning

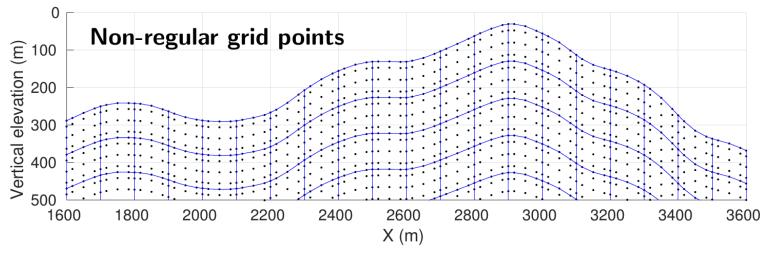


Prior information? $L_w = 25 \text{ m}$ and L_u , $L_v = 25 \sim 100 \text{ m}$; Dip & azimuth from true models.

Preconditioning for FE mesh



	Accuracy	Efficiency	Nonstationarity
Projection between SEM & Cartesian meshes	?	?	?
Explicit truncated convolution $\underline{s}(\underline{x}) \approx B_{3D}(\underline{x}) *_{\underline{\Omega}_{r}} \underbrace{g(\underline{x})}_{Raw}$ SmoothedRawgradientgradient	\checkmark	?	\checkmark
Bessel smoothing $B_{3D}^{-1}(\mathbf{x}) * \underbrace{\mathbf{s}(\mathbf{x})}_{\text{Smoothed}} = \underbrace{\mathbf{g}(\mathbf{x})}_{\text{Raw}}$ gradient gradient	\checkmark	\checkmark	\checkmark





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3D elastic example: subset of SEAM II

2

3

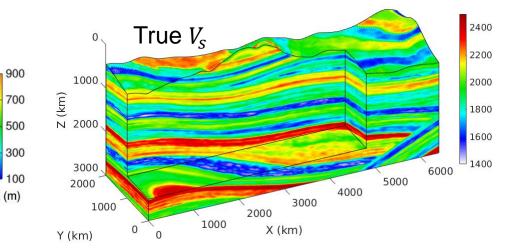
X (km)

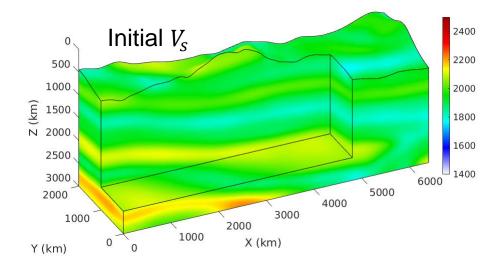


Significant topography variation: $\Delta Z \approx 800$ m.

۲ (km)

- 3D surface acquisition:
 - \geq 4 × 20 sources
 - $\Delta S_{\chi} = 320 \text{ m}$ $\Delta S_v = 500 \text{ m}$
 - ➢ 82600 receivers, 12.5m, 3C
- S(t) =Ricker wavelet centered at 3.5 Hz
- **Meshing**: P4 high-order topography representation.
- Initial models = Smoothed version of true model.
- **Simultaneous** inversion for V_p and V_s
- Smoothed density is kept unchanged.
- 60 FWI iterations using the I-BFGS optimization method.



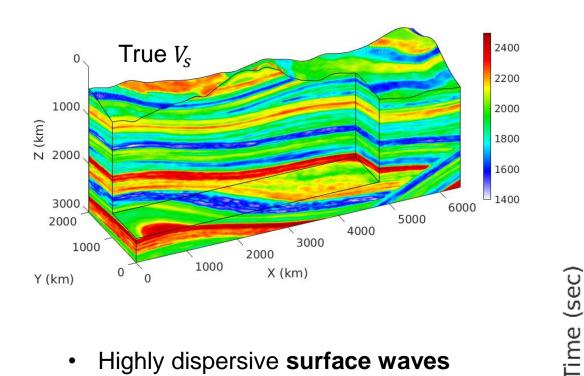


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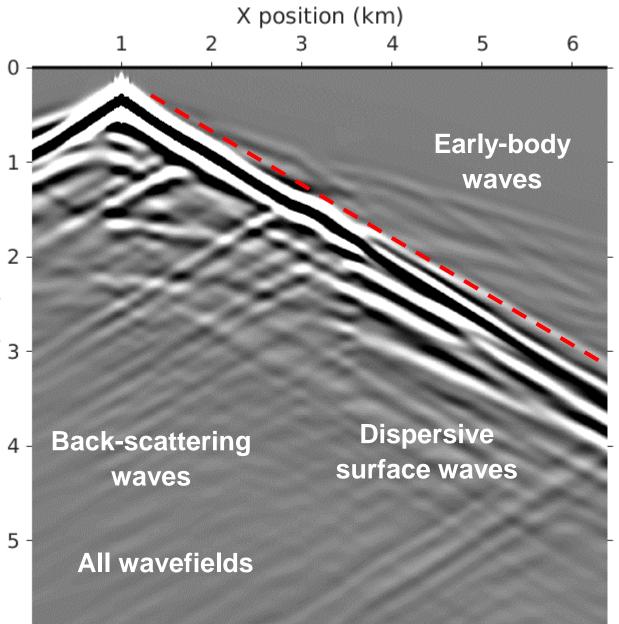
Z (m)

Complex wavefield



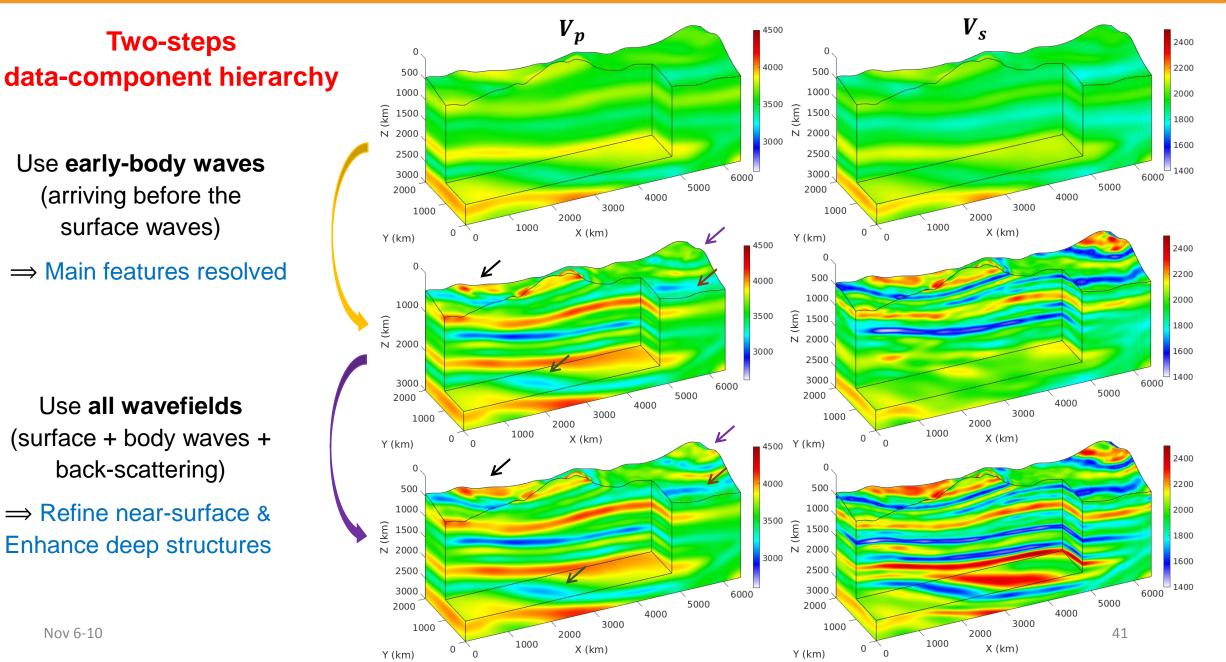


- Highly dispersive surface waves
- Waves conversion P-S, bodysurface
- Back-scattering due to steep-slope • at the surface.



Simple FWI data-driven strategy

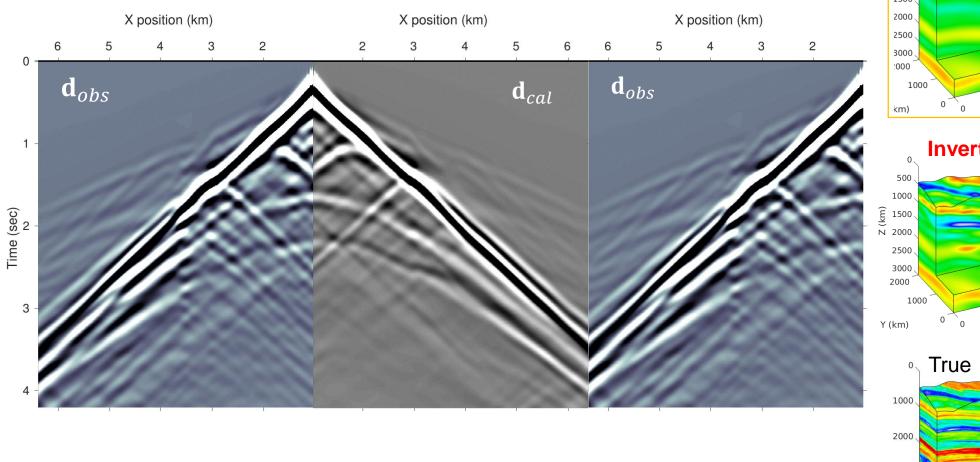


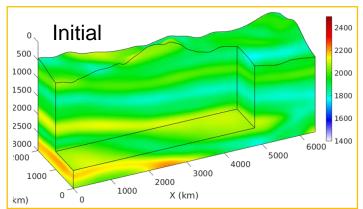


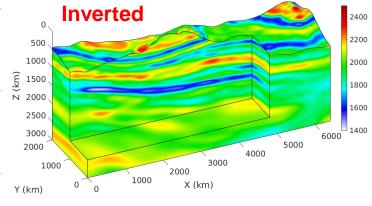
Data comparison

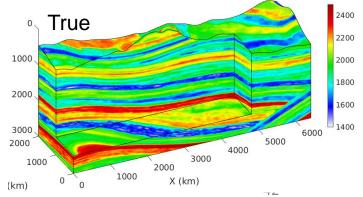


0. Initial models





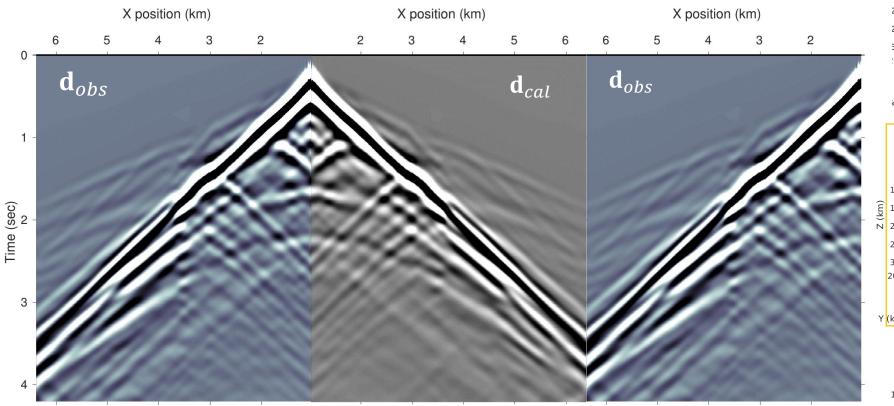


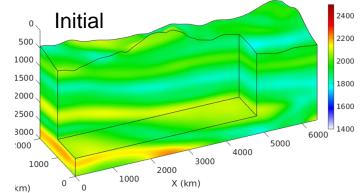


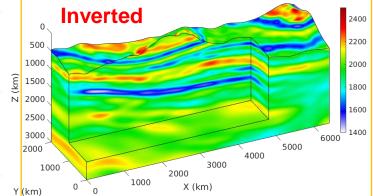
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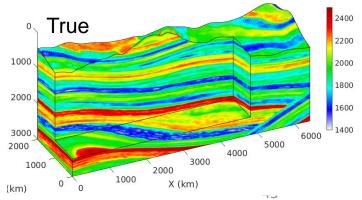








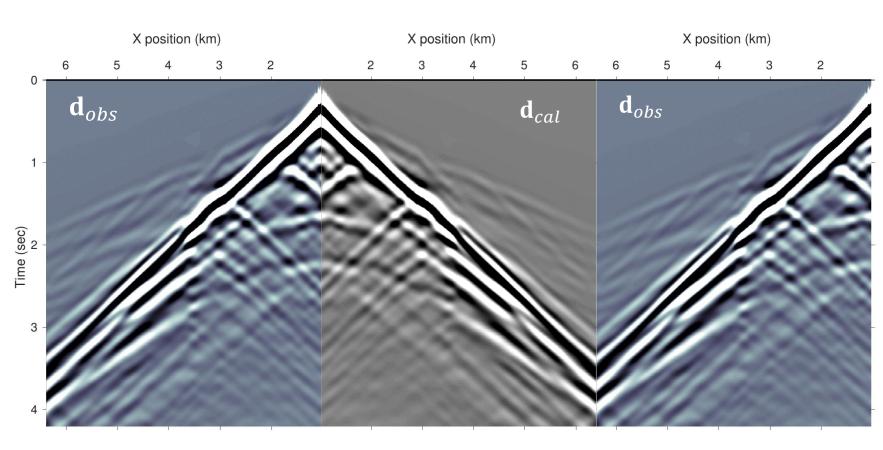


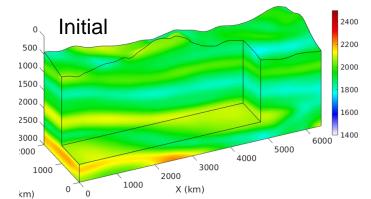


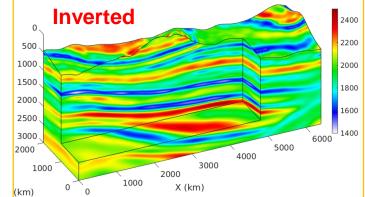
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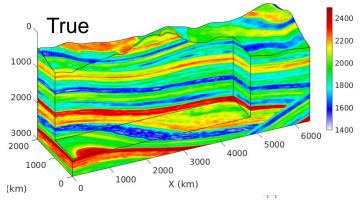


2. FWI with all wavefields









Numerical efficiency

From the 3D elastic example

- Deformed mesh: $32 \times 68 \times 28$ elements ($129 \times 273 \times 113$ dofs).
- 6 sec recording time (10 000 time-steps).
- 1600 cores (20 cores/shot)

	Memory estimation	Elapsed time 1 st gradient	Elapsed time 60 FWI iterations
Elastic	44 Gb /shot	20 min	20.8 h
Viscoelastic	74 Gb /shot	1.2 h	75 h



Extrapolation for viscoelastic case

- 80 checkpoints for incident wavefield reconstruction.
- Recomputation ratio \approx 3.

Conclusion I



- Moving to 3D visco-aniso-elastodynamics FWI is now possible for crustal land data (PhD topic of P.T. Trinh).
- > Application to real datasets: multi-parameters images?
- Which macro-scale parameters are important for meso-scale downscaling investigation for micro-scale interpretation:

Q attenuation factor is important!

Cautiousness in interpretation as FWI results seem often quite realistic.

Conclusion II



- Different families: what is the « best » set ???
 - Velocity slowness-square of slowness
 - Density- Buoyancy-Impedance
 - Attenuation-Inverse of attenuation
 - Log; tanh (or any non-linear transform) ...
- Hints: mitigate the leakage between parameters ...

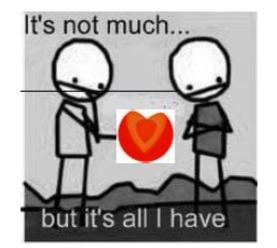


Model parameters # inference parameters # physical parameters ...
 FWI reconstructs model parameters ... at the macro-scale level ...



Thank you very much!

 $FWI = \lambda/2$



> Cycle skipping problem: under control.

- > Local minima issue: better mitigation.
- > Multiple parameter issue: important for apps.

Do not forget the UQ guy!